Optimal Control of Quadrotor Flight for Aircraft Inspection

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**Quadrotor Unmanned Aerial Vehicle (UAV) has a variety of applications in industry and domestic environments, aerial inspection for large structure is an example which requires the quadrotor to fly around a target structure and take photos with the equipped camera. However, this becomes challenge when the quadrotor is required to autonomously fly around an aircraft in a GPS denied environment such as MRO shop. Some sensing technologies such as RGBD sensor need to be used for avoiding obstacles during aircraft inspection by a quadrotor. This work presents an approach of using quadrotor with a light-weight consumer-grade RGBD sensor for aircraft inspection, focusing on the optimal trajectory control from one inspection point to another. The environment is modeled with the point cloud captured by the RGBD sensor and gridded in octree voxels with specified resolution, then the free space for quadrotor to fly can be computed. With optimizing the energy of the flight, the optimal trajectory can be obtained.**

1. **Nomenclature**

= mass of the quadrotor

= gravity constant

= length between rotor and the center of gravity of the quadrotor

= thrust generated by th rotor

= drag generated by th rotor

= rotor speed of th rotor

= thrust factor and Drag factor correspondingly

= position of the quadrotor in inertial frame

= roll, pitch yaw of the quadrotor correspondingly

= linear velocity of the quadrotor in body frame

= angular rate of the quadrotor in body frame

= moment of inertia along and axes correspondingly

= co-state of Hamiltonian system

= performance index

= starting time and ending time of quadrotor flying

1. **Introduction**

The quadrotor UAV is a helicopter with four rotors typically designed in a cross configuration. One pair of opposite

rotors rotate clockwise, and the other pair of rotors rotate counterclockwise to balance the torque. The attitude and altitude of the quadrotor can be controlled by changing the thrusts of the rotors. The quadrotor has been used in aerial photography for mapping, inspection and traffic monitoring in urban area, search and rescue operation for missing persons and natural disasters. And It is also a great platform for control systems research as its non-linear nature and under-actuated configuration [1].

The advance in sensing technology make the large structure and scene reconstruction possible due to the efficient method of acquiring 3D model of the structure and the scene [2]. This further contributes to the development of technologies such as simultaneously localization and mapping (SLAM). The development of lightweight and low-cost 3D RGBD sensor, such as Intel RealSense, makes the UAV inspection a potential application to aircraft maintenance, repair, and overhaul (MRO). Airbus used a quadrotor with equipped a camera for aircraft inspection [3], however the system is controlled by a human operator, the efficiency of inspecting whole aircraft is improved but still time-consuming because of that the shape of the aircraft is complex and inspection path is complicated. Therefore, an autonomous quadrotor inspection system is necessary for the aircraft MRO for improving the efficiency and saving the labor cost. And one of the problems need to be addressed is controlling the quadrotor from one inspection point to another while avoiding the obstacles and achieve some level of optimization such as minimum energy.

1. **Dynamics Modeling**

A general quadrotor system [4] with control inputs is shown in Fig. 1. The quadrotor (and its propellers) is assumed to be rigid and symmetric with respect to the and axes in this work, the center of gravity and the body fixed frame origin are assumed to be coincident. In addition, thrust and drag are proportional to the square of propeller’s speed.

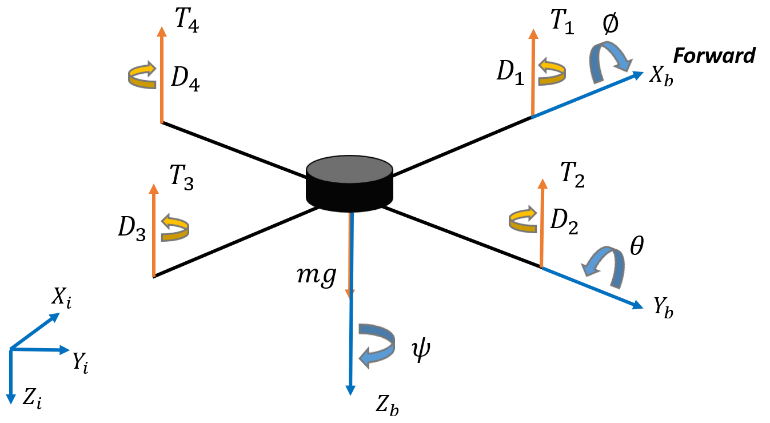


Fig. 1 Free body diagram of a quadrotor

The attitude and position of the quadrotor can be controlled to desired values by changing the speeds of the four motors. The thrusts are then generated by rotors rotation, the pitching moment and rolling moment are caused by the difference of four rotors thrusts, the gravity and the yawing moment which is caused by the unbalanced of the four rotors rotational speeds. The relationship between rotors speeds and generated thrusts and torques is given by

(1)

where, and are thrust and rotational torque acting on th rotor correspondingly, which are determined by thrust factor , drag factor and the distance from the rotor to the center of gravity. From Equation (1), we can determine the rotor speed by multiplying the inverse matrix of thrust factor and drag factor by the control input which can be obtained by using optimal control method.

By defining a generalized coordinates vector , containing the linear and angular position of the quadrotor in the inertial frame and a vector , containing the linear and angular velocities in the body frame, with the transformation between the inertia frame and body frame as following:

(2)

where , , etc., the kinematic model of the quadrotors is given by

(3)

The dynamics model of the quadrotor in body frame without considering the external disturbance and the gyroscopic moments is given by

(4)

where, is the mass of the quadrotor, is the gravity constant, , and are the moments of inertial about , and axes correspondingly. Thus, the dynamics equations of the quadrotor in state space with a control input vector and a state vector is given by

(5)

1. **Optimal Control**

The goal of this work is to find control inputs to minimize a control energy of that a quadrotor flies from one inspection point to another. The performance index is given by:

(6)

subjecting to the dynamics of quadrotor as Equation (5) presents. and T are the starting time and the ending time correspondingly. The problem can be modeled as a two-point boundary value problem (TPBVP) with a given initial state , , ,, , and a final state , , ,, , . By introducing co-state vector , the Hamiltonian function of the system is formed as

(7)

The necessary conditions are given by

(8)

(9)

where the costate conditions are given in details as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

And the stationary condition is given by

(11)

The optimal control is then determined by Equation (11), which is a closed-loop feedback control, depending on the states and costate.

The boundary conditions are the initial condition , the final condition correspondingly, the final constraint , and general boundary conditions:

(12)

where is the final state portion of the performance index, which is zero in this work, is a Lagrange multiplier for the final state constraints. As the final state is given; by giving a fixed flying time interval, which means , the problem is formed as a fixed final state and fixed final time problem. There are 24 differential equations to solve, 12 for the states and 12 for the co-states. There are 24 boundary conditions can be used for solving the problem. The problem can be solved by numerical method “bvp5c” using Matlab software [5].

1. **Simulation**

The system parameters are given in Table. 1 for simulation in Matlab.

**Table. 1 Parameters of quadrotor for simulation**

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Parameter** | **Value** | **Units** |
| Mass |  | 1.56 |  |
| Gravitational Acceleration |  | 9.8 |  |
| Moment of inertia about x axis |  | 0.1147 |  |
| Moment of inertia about x axis |  | 0.0576 |  |
| Moment of inertia about x axis |  | 0.1712 |  |
| Distance from rotor to center of gravity |  | 0.2 |  |
| Trust factor |  | 3.8305 |  |
| Drag factor |  | 2.2518 |  |

## Case 1: Vertical Ascending

The first case is vertical ascending from to inspection point in 10 seconds (). The optimal flying path is a “straight” vertical line as shown in Fig. 2 a), the state, co-state, control input, motor speed and performance index relative to time are shown in Fig. 2 b), Fig. 2 c), Fig. 2 d), Fig. 2 e) and Fig. 2 f) correspondingly.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| Fig. 2 a) Optimal path | Fig. 2 b) States relative to time | Fig. 2 c) Co-states relative to time |
|  |  |  |
| Fig. 2 d) Control input over time | Fig. 2 e) Motor speed over time | Fig. 2 f) Performance over time |

## Case 2: Vertical Ascending and Yawing

The second case is vertical ascending and yawing from to inspection point in 10 seconds (). The optimal flying path is remaining a “straight” vertical line as shown in Fig. 3 a), the state, co-state, control input, motor speed and performance index relative to time are shown in Fig. 3.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| Fig. 3 a) Optimal path | Fig. 3 b) States relative to time | Fig. 3 c) Co-states relative to time |
|  |  |  |
| Fig. 3 d) Control inputs over time | Fig. 3 e) Motor speed over time | Fig. 3 f) Performance over time |

## Case 3: Descending and Yawing

The third case is a descending and yawing from to inspection point in 10 seconds (). The optimal flying path is shown in Fig. 4 a), and the state, co-state, control input, motor speed and performance index relative to time are shown in Fig. 4.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| Fig. 4 a) Optimal path | Fig. 4 b) States relative to time | Fig. 4 c) Co-states relative to time |
|  |  |  |
| Fig. 4 d) Control input over time | Fig. 4 e) Motor speed over time | Fig. 4 f) Performance over time |

1. **Discussion**

By using Matlab algorithm “bvp5c” to solve the boundary value problem, an initial guess (co-states values in this work) of the solution to a BVP must be provided. As the BVP problem can have no solution, a finite number of solutions or infinitely many solutions, the quality of initial guess can be critical for the solver performance and even for a successful computation. Therefore, such method cannot be applied to the online optimal control of aircraft inspection task, but an offline planning can be executed if all inspection points are known in advance.

A more useful task of optimal control in aircraft inspection is to find a control sequence to minimize the time of flying from one inspection to another while under saturated control input. This will be one of my future works.

**References**

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